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DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF MATERIALS AS FUNCTIONS OF TEMPERATURE

G. A. Surkov

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DETERMINATION OF THERMOPHYSICAL CHARACTERISTICS OF MATERIALS AS FUNCTIONS OF TEMPERATURE

G. A. Surkov[1]

ABSTRACT

The paper attempts to determine thermophysical characteristics as functions of temperature.

Let us consider a symmetrical body (the treatment for asymmetrical bodies is similar). /22*

Assuming the density ρ of the body to be independent of temperature, and the thermal conductivity λ and heat capacity c to be dependent on the space coordinate and time, the problem can be mathematically formulated as follows

$$\frac{\partial T(N,\tau)}{\partial \tau} = \left(B(\tau) + \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} b_{mp} N^{m-p} \tau^{p} \right) \left(\frac{\partial^2 T(N,\tau)}{\partial N^2} + \frac{k-1}{N} \frac{\partial T(N,\tau)}{\partial N} \right) + \tag{1}$$

$$+ \left(A(\tau) + \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} a_{mp} N^{m-p} \tau^{p} \right) \frac{\partial T(N,\tau)}{\partial N} \quad (k = 1, 2, 3),$$

$$T(N,\tau)|_{\tau=0} = T_0, \tag{2}$$

$$\left. \frac{\partial T(N,\tau)}{\partial N} \right|_{N=0} = 0, \tag{3}$$

*Numbers given in margin indicate pagination in original foreign text.

[1]Presented by A. V. Lykov, Member of the BSSR Academy of Sciences.

$$\left.\frac{\partial T(N,\tau)}{\partial N}\right|_{N=R}=\frac{q(\tau)}{\lambda(R,\tau)}, \tag{4}$$

where

$$\frac{\lambda(N,\tau)}{\rho c(N,\tau)}=B(\tau)+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}b_{mp}N^{m-p}\tau^{p}, \tag{5}$$

$$\frac{\lambda_N'(N,\tau)}{\rho c(N,\tau)}=A(\tau)+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}a_{mp}N^{m-p}\tau^{p}. \tag{6}$$

In addition, let us assume (this can always be done experimentally) that the temperatures are given at certain points, i.e.,

$$T(N,\tau)|_{N=0}=F_{(0)}(\tau), \tag{7}$$

$$T(N,\tau)|_{N=R/2}=F_{(R/2)}(\tau), \tag{8}$$

$$T(N,\tau)|_{N=R}=F_{(R)}(\tau). \tag{9}$$

From the known functions (7-9) we can readily construct the temperature field. We shall seek it in the form

$$T(N,\tau)=a(\tau)N^{3}+b(\tau)N^{2}+F_{(0)}(\tau). \tag{10}$$

Using conditions (7-9), we obtain an expression for the temperature field

$$\begin{aligned}T(N,\tau)=\Phi(N,\tau)=\frac{2}{R^{3}}\left(F_{(R)}(\tau)-4F_{(R/2)}(\tau)+3F_{(0)}(\tau)\right)N^{3}+\\+\frac{1}{R^{2}}\left(8F_{(R/2)}(\tau)-F_{(R)}(\tau)-7F_{(0)}(\tau)\right)N^{2}+F_{(0)}(\tau).\end{aligned} \tag{11}$$

From equation (11), we have

$$\frac{\partial T(N,\tau)}{\partial N} = \Phi'_N(N,\tau), \tag{12}$$

$$\frac{\partial^2 T(N,\tau)}{\partial N^2} = \Phi''_{NN}(N,\tau). \tag{13}$$

Substituting (12) and (13) into the right-hand part of equation (1), we obtain

$$\frac{\partial T(N,\tau)}{\partial \tau} = \left(B(\tau) + \sum_{m=1}^{\infty}\sum_{p=0}^{m-1} b_{mp} N^{m-p}\tau^{p}\right)(\Phi''_{NN}(N,\tau) + \overline{\Phi}'_N(N,\tau)) + \\ + \left(A(\tau) + \sum_{m=1}^{\infty}\sum_{p=0}^{m-1} a_{mp} N^{m-p}\tau^{p}\right)\Phi'_N(N,\tau), \tag{14}$$

where $\overline{\Phi}'_N(N,\tau) = \frac{k-1}{N}\Phi'_N(N,\tau).$

Using boundary conditions (7) and (9), we obtain a system of two equations for determining functions $A(\tau)$ and $B(\tau)$, i.e.,

$$F'_{(0)\tau}(\tau) = B(\tau)(\Phi''_{NN}(0,\tau) + \overline{\Phi}'_N(0,\tau)),$$

$$F'_{(R)\tau}(\tau) = \left(B(\tau) + \sum_{m=1}^{\infty}\sum_{p=0}^{m-1} b_{mp} R^{m-p}\tau^{p}\right)(\Phi''_{NN}(R,\tau) + \overline{\Phi}'_N + (R,\tau)) + \tag{15}$$

$$+ \left(A(\tau) + \sum_{m=1}^{\infty}\sum_{p=0}^{m-1} a_{mp} R^{m-p}\tau^{p}\right)\Phi'_N(R,\tau), \tag{16}$$

whence we have

$$A(\tau) = \Psi_{A_1}(\tau) - \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} a_{mp} R^{m-p} \tau^p - \Psi_{A_2} \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} b_{mp} R^{m-p} \tau^p, \tag{17}$$

$$B(\tau) = \Psi_{B_1}(\tau), \tag{18}$$

where

$$\Psi_{A_1}(\tau) = \frac{F'_{(R)\tau}(\tau)(\Phi''_{NN}(0,\tau) + \overline{\Phi}'_N(0,\tau)) + F'_{(0)\tau}(\tau)(\Phi''_{NN}(R,\tau) + \overline{\Phi}_N(R,\tau))}{\Phi'_N(R,\tau)(\Phi''_{NN}(0,\tau) + \overline{\Phi}'_N(0,\tau))}, \tag{19}$$

$$\Psi_{A_2}(\tau) = \frac{\Phi''_{NN}(R,\tau) + \overline{\Phi}'_N(R,\tau)}{\Phi'_N(R,\tau)}, \tag{20}$$

$$\Psi_{B_1}(\tau) = \frac{F'_{(0)\tau}(\tau)}{\Phi''_{NN}(0,\tau) + \overline{\Phi}'_N(0,\tau)}. \tag{21}$$

Substituting the values of functions A(τ) and B(τ) into (14) and solving the ordinary differential equation with initial condition (2), we shall have

$$\begin{aligned} T(N,\tau) = T_0 + \int_0^{\tau} \Big[\Big(\Psi_{B_1}(\tau) + \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} b_{mp} N^{m-p} \tau^p \Big)(\Phi''_{NN}(N,\tau) + \\ + \overline{\Phi}'_N(N,\tau)) + \Big(\Psi_{A_1}(\tau) - \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} a_{mp} R^{m-p} \tau^p - \Psi_{A_2} \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} b_{mp} R^{m-p} \tau^p + \\ + \sum_{m=1}^{\infty} \sum_{p=0}^{m-1} a_{mp} N^{m-p} \tau^p \Big) \Phi'_N(N,\tau) \Big] d\tau. \end{aligned} \tag{22}$$

After integrating, we can represent expression (22) in the form

$$T(N,\tau)=f(N,\tau)-\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}a_{mp}\tau^{p}(R^{m-p}\varphi_{mp}(N,\tau)-N^{m-p}\psi_{mp}(N,\tau))-$$
$$-\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}b_{mp}\tau^{p}(R^{m-p}\xi_{mp}(N,\tau)-N^{m-p}\eta_{mp}(N,\tau)). \tag{23}$$

The unknown coefficients will be determined as follows.

We take function $\Phi(N, \tau)$ at point $N = R/2$ at instants of time τ_j ($j = 1, 2, \ldots 2m$) and set it equal to the right-hand part of equation (23) at the corresponding point. We then obtain a system of 2m algebraic equations for determining a_{mp} and b_{mp}

$$\sum_{j=1}^{\infty}\Phi(R/2,\tau_j)=f(R/2,\tau_j)-\sum_{j=1}^{\infty}\Big[\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}a_{mp}\tau_j^{p}(R^{m-p}\varphi_{mp}(R/2,\tau_j)-$$
$$-(R/2)^{m-p}\psi_{mp}(R/2,\tau_j))+$$
$$+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}b_{mp}\tau_j^{p}(R^{m-p}\xi_{mp}(R/2,\tau_j)-(R/2)^{m-p}\eta_{mp}(R/2,\tau_j))\Big]. \tag{24}$$

Substituting the known values of b_{mp} into equation (5) and considering (21), we obtain the value of the thermal diffusivity at point R

$$a(R,\tau)=\frac{F'_{(0)\,\tau}(\tau)}{\Phi''_{NN}(0,\tau)+\overline{\Phi}_{N}(0,\tau)}+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}b_{mp}R^{m-p}\tau^{p}. \tag{25}$$

The value of the thermal conductivity will be determined from condition (4) and expression (11)

$$\lambda(R,\tau)=\frac{q(\tau)}{\Phi'_{N}(R,\tau)}. \tag{26}$$

In the case where the value of the thermal flux $q(\tau)$ is unknown, the thermal conductivity coefficient at point R can be determined as follows. From (5) and (6), excluding $\rho c(N,\tau)$, we have

$$\frac{\lambda_N'(N,\tau)}{\lambda(N,\tau)}=\frac{Q(N,\tau)}{P(N,\tau)}, \tag{27}$$

where

$$Q(N,\tau)=A(\tau)+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}a_{mp}N^{m-p}\tau^p, \tag{28}$$

$$P(N,\tau)=B(\tau)+\sum_{m=1}^{\infty}\sum_{p=0}^{m-1}b_{mp}N^{m-p}\tau^p. \tag{29}$$

Let τ_0 be the time lag of the action of the thermal flux at point N = 0. Then, integrating (27) with respect to N, for $\tau = \tau_0$ we have

$$\lambda_1(N,\tau_0)=\lambda_0\exp\left[\int\frac{Q(N,\tau_0)}{P(N,\tau_0)}\,dN\right], \tag{30}$$

where λ_0 is the thermal conductivity coefficient at the initial temperature and is assumed to be known.

From (30) we can readily obtain the values of λ_1 at point R for $\tau = \tau_0$, i.e.,

$$\lambda_1(R,\tau_0)=\lambda_0\exp\left[\frac{Q(N,\tau_0)}{P(N,\tau_0)}dN\right]\Bigg|_{N=R}. \tag{31}$$

The temperature at point N = 0, equal to the temperature $T(R, \tau_0)$, will reach (?) at instant τ_1. Integrating (27) with respect to N, at instant $\tau = \tau$ we obtain $\lambda_2(R, \tau_1)$, i.e.,

$$\lambda_2(R, \tau_1) = \lambda_1 \exp\left[\int \frac{Q(N, \tau_1)}{P(N, \tau_1)} dN\right]\Bigg|_{N=R}. \tag{32}$$

Continuing, we shall similarly have

$$\lambda_N(R, \tau_{n-1}) = \lambda_{n-1} \exp\left[\int \frac{Q(N, \tau_{n-1})}{P(N, \tau_{n-1})} dN\right]\Bigg|_{N=R}. \tag{33}$$

Thus, from (30-33) we can obtain a precise dependence of the thermal conductivity coefficient on the temperature.

As usual, the value of the heat capacity will be

$$c(R, \tau) = \frac{\lambda(R, \tau)}{\rho a(R, \tau)}. \tag{34}$$

In order to obtain the dependence of the thermal diffusivity and thermal capacity on the temperature, we break up the time segment $(0, \tau_k)$ into n intervals $\Delta\tau_n$. Taking the values of the thermophysical characteristics at instants τ_n and correspondingly the values of the temperature $\Phi(R, \tau_n)$, we plot the curves which will express their dependence on the temperature.

Institute of Heat and Mass Transfer,
Academy of Sciences BSSR

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